

Bond-energy variables for Ising spin-glass dynamics

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We point out the advantages of computing the dynamics of Ising spin glasses and related (e.g., bond or site dilution) systems in zero external field by using bond-energy variables $b_{ij} = s_i s_j J_{ij}$ instead of the usual spin variables and coupling constants.

In the absence of an external field, the energy of a spin glass is invariant with respect to Mattis transformations (flipping any spin and all its coupling constants). The use of bond-energy variables, in place of the usual spin variables and coupling constants, projects out this trivial symmetry, reducing by N bits the amount of memory required to simulate an N spin system, since the evolution of the system can be computed from the bond-energy variables alone, without explicit storage of the spins or coupling constants (cf. Fig. 1). The complexity of the updating operating itself is reduced, regardless of whether one uses a traditional heat-bath Monte Carlo algorithm¹ or newer constant-energy deterministic methods,² since the decision whether to flip a spin depends simply on the sum of the bond energies at that site, rather than on a sum of products of spins by coupling constants as in the usual representation. The number of memory accesses is also slightly reduced, an update in the usual representation requiring $2z + 1$ fetches (central spin, z neighbors, and z coupling constants) and 1 store (the new value of the central spin),

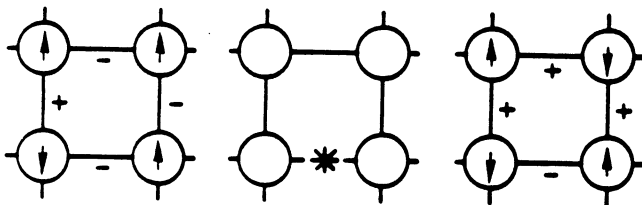


FIG. 1. Part of a typical Ising spin glass is shown in the usual representation (left) in terms of spins and coupling constants, and in the bond-energy representation (center). Three of the bonds are in the ground state and one is excited (asterisk). On the right is another spin-glass configuration sharing the same bond-energy representation. It is obtained from the configuration on the left by flipping the upper right spin together with all its coupling constants.

while in the bond energy representation it requires z fetches (old bond energies) and z stores (new bond energies, opposite in sign to the old if a flip has occurred). Moreover, this balanced pattern of accesses lends itself to exploitation by special-purpose hardware, which can, for example, achieve a reduction in addressing circuitry setup time when a store is performed at the same location as the preceding fetch.

So long as the system's Hamiltonian is Mattis invariant, its dynamics can be accurately simulated in the bond-energy representation. The evolution of non-Mattis-invariant quantities such as magnetization can also be followed, but this is best done off-line, by periodically generating a snapshot of the system's spin configuration from the corresponding bond energy configuration and the known, fixed coupling constants. The bulk of the computing effort would still be in the on-line calculation serving to evolve the bond-energy configuration from one snapshot to the next.

Aside from numerical advantages, the bond-energy representation has conceptual advantages. While removing the irrelevant Mattis degrees of freedom, it introduces other degrees of freedom with the result that frustration is explicitly represented within the dynamical variables of the model, rather than being imposed by external parameters (coupling constants). Thus it is possible, without using extra variables, to treat systems (e.g., string models³) in which the frustration undergoes its own deterministic or stochastic evolution, rather than being frozen in as in a spin glass.

Finally, bond-energy variables reveal an analogy between deterministic Ising models and another major class of cellular automata relevant to physics, viz., particle-conserving lattice gases.⁴ The excited bonds in a bond-energy representation can be thought of as "particles," and the deterministic Ising dynamics may be thought of as a collision rule for these particles.

¹See e.g., J. H. Condon and A. T. Ogielski, *Rev. Sci. Instrum.* **56**, 1691 (1985).

²G. Vichniac, *Physica D* **10**, 96 (1984); Y. Pomeau, *J. Phys. A* **17**, L415 (1984); M. Creutz, *Ann. Phys. (N.Y.)* **167**, 62 (1986); H. Hermann, *Saclay Report No. 86-060*, 1986 (un-

published).

³D. H. Lee and G. Grinstein *Phys. Rev. Lett.* **55**, 541 (1985).

⁴U. Frisch, B. Hasslacher, and Y. Pomeau, *Phys. Rev. Lett.* **56**, 1505 (1986); L. Kadanoff, *Phys. Today* **39** (No. 9), 7 (1986).