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Notes on Quantum and Classical State Transposition: Resource Bounds for Coding Mixed States

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(June 23, 1995)

Abstract

We consider the problem of "transmitting" or "transposing" mixed classical or quantum states. We distinguish several possible definitions of transposition, depending on whether information about the particular state prepared is used in the encoding process, and also on the degree to which transposition process is required to preserve correlations with other systems. For several of these definitions, we give examples in which the required resources are less than $S(\rho_{av})$, the entropy of the average distribution or density operator.

I. INTRODUCTION

The emergence of quantum cryptography and quantum computation as burgeoning fields of research highlights the need to understand the process of storing and transmitting quantum states. A fundamental question is: what are the minimal resources required to store or transmit quantum states from a given ensemble? Jozsa and Schumacher [1] have proved the "Quantum Noiseless Coding Theorem" for pure quantum states. It says that for an ensemble of pure states $|\psi_i\rangle$ with probabilities p_i, the "quantum channel capacity" required is $S(\rho)$, the Von Neumann entropy of the density operator of the ensemble, $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$. Block coding is used, and the bound is approached in the large-N limit. The quantum channel capacity is measured in qubits, where a qubit is the storage capacity of a two-state quantum system. The channel capacity can be thought of as the base-two log of the number of required Hilbert space dimensions. Jozsa and Schumacher and others [3] are at work on a generalization of this theorem which will apply to ensembles of mixed states. The attempt to make such a generalization raises the fundamental question of what it means to transmit (or *transpose*, to use Schumacher's preferred term) a state, when that state contains a subjective element (probabilities). The question can be investigated in both quantum and classical contexts. There are different answers to this question which we argue will lead to different bounds on required channel capacity.